

Corrigé Exercice 11

1. a) On compare les deux calculs $f(x) - g(x)$ et $x(x - 7)$:

A gauche, on a : $\begin{aligned} f(x) - g(x) &= (3 - 5x) - (2x + 3 - x^2) \\ &= 3 - 5x - 2x - 3 + x^2 \\ &= -7x + x^2 \end{aligned}$	A droite, on a : $\begin{aligned} x(x - 7) &= x^2 - 7x \\ &= -7x + x^2 \end{aligned}$ aussi
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Donc on a bien : $f(x) - g(x) = x(x - 7)$.

b) On doit comparer C et le calcul $-x^2 - 1,3x + 5$:

A gauche, on a : $\begin{aligned} C = R - B \\ &= 0,7x - (2x + x^2 - 5) \\ &= 0,7x - 2x - x^2 + 5 \\ &= -1,3x - x^2 + 5 \end{aligned}$	A droite, on a : $\begin{aligned} -x^2 - 1,3x + 5 \\ &= -1,3x - x^2 + 5 \end{aligned}$ aussi
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Donc on a bien : $C = -x^2 - 1,3x + 5$.

2. a)

A gauche, on a : $h'(x) = -x^2 + 3x - 2$	A droite, on a : $\begin{aligned} (x - 1)(2 - x) \\ &= 2x - x^2 - 2 + x \\ &= -x^2 + 3x - 2 \end{aligned}$ aussi
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Donc on a bien : $h'(x) = (x - 1)(2 - x)$.

b)

A gauche, on a : $\begin{aligned} K = 2(t^2 + 3t) - 20 \\ &= 2t^2 + 6t - 20 \end{aligned}$	A droite, on a : $\begin{aligned} 2(t + 5)(t - 2) \\ &= (2t + 10)(t - 2) \\ &= 2t^2 - 4t + 10t - 20 \\ &= 2t^2 + 6t - 20 \end{aligned}$ aussi
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Donc on a bien : $K = 2(t + 5)(t - 2)$.

c) A gauche, on a :

$$f(x) = -3x(x + 8) - 21 = -3x^2 - 24x - 21$$

A droite, on a :

$$\begin{aligned} -3(x + 1)(x + 7) &= (-3x - 3)(x + 7) \\ &= -3x^2 - 21x - 3x - 21 \\ &= -3x^2 - 24x - 21 \end{aligned} \quad \text{aussi}$$

Donc on a bien : $f(x) = -3(x + 1)(x + 7)$.

c) On compare les deux calculs $g(x) - f(x)$ et $-5(x + 1)$:

A gauche, on a : $\begin{aligned} g(x) - f(x) \\ &= (-8 + 2x) - (7x - 3) \\ &= -8 + 2x - 7x + 3 \\ &= -5 - 5x \end{aligned}$	A droite, on a : $\begin{aligned} -5(x + 1) \\ &= -5x - 5 \end{aligned}$
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Donc on a bien : $g(x) - f(x) = -5(x + 1)$.

d) On doit comparer B et le calcul $0,5(x - 1)(x - 4)$:

A gauche, on a : $\begin{aligned} B = R - C \\ &= 2x - (4,5x - 0,5x^2 - 2) \\ &= 2x - 4,5x + 0,5x^2 + 2 \\ &= 0,5x^2 - 2,5x + 2 \end{aligned}$	A droite, on a : $\begin{aligned} 0,5(x - 1)(x - 4) \\ &= 0,5x^2 - 2x - 0,5x + 2 \\ &= 0,5x^2 - 2,5x + 2 \end{aligned}$
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Donc on a bien : $B = 0,5(x - 1)(x - 4)$.

d) On a d'abord :

$$f'(x) = 2(x^2 - x - 2) = 2x^2 - 2x - 4$$

On a ensuite :

$$\begin{aligned} (2x + 2)(x - 2) &= 2x^2 - 4x + 2x - 4 \\ &= 2x^2 - 2x - 4 \end{aligned}$$

On a donc bien : $f'(x) = (2x + 2)(x - 2)$

e) On a d'abord :

$$\begin{aligned} K = 12 - 3(a - 1)^2 &= 12 - 3(a^2 - 2a + 1) \\ &= 12 - 3a^2 + 6a - 3 = 9 - 3a^2 + 6a \end{aligned}$$

On a ensuite :

$$\begin{aligned} 3(3 - a)(a + 1) &= 9a + 9 - 3a^2 - 3a \\ &= 9 - 3a^2 + 6a \end{aligned}$$

On a donc bien : $K = 3(3 - a)(a + 1)$.

f) On a $b(x) = 4\left(x - \frac{1}{2}\right)^2 = 4\left(x^2 - x + \frac{1}{4}\right)$
 $= 4x^2 - 4x + 1$

CQFD !

Corrigé Exercice 12

$$\begin{aligned}
 1) : A &= (4x - 12)(x - 3) - (2x - 6)^2 \\
 A &= (4x^2 - 12x - 12x + 36) - (4x^2 - 24x + 36) \\
 A &= 4x^2 - 24x + 36 - 4x^2 + 24x - 36 \\
 A &= 0
 \end{aligned}$$

CQFD

2) a) On a d'unepart : $f(x) = 3x^2 + 12x - 15$

Et d'autre part :

$$\begin{aligned}
 3(x + 2)^2 - 27 &= 3(x^2 + 4x + 4) - 27 \\
 &= 3x^2 + 12x + 12 - 27 \\
 &= 3x^2 + 12x - 15
 \end{aligned}$$

On a donc bien $f(x) = 3(x + 2)^2 - 27$

b) On a : $f(x) = 3x^2 + 12x - 15$

$$\begin{aligned}
 \text{Et : } 3(x - 1)(x + 5) &= 3(x^2 + 5x - x - 5) \\
 &= 3(x^2 + 4x - 5) \\
 &= 3x^2 + 12x - 15
 \end{aligned}$$

On a donc bien : $f(x) = 3(x - 1)(x + 5)$

$$\begin{aligned}
 3) A &= (x + 8)^2 - 50 = x^2 + 16x + 64 - 50 \\
 &= x^2 + 16x + 14
 \end{aligned}$$

$$\begin{aligned}
 \text{et } B &= 2(x + 3)^2 - (x - 2)^2 \\
 &= 2(x^2 + 6x + 9) - (x^2 - 4x + 4) \\
 &= 2x^2 + 12x + 18 - x^2 + 4x - 4 \\
 &= x^2 + 16x + 14
 \end{aligned}$$

On a bien $A = B$

$$\begin{aligned}
 4) \text{On a : } E &= 3x^2 - (3x + 6)(x - 2) \\
 &= 3x^2 - (3x^2 - 6x + 6x - 12) \\
 &= 3x^2 - (3x^2 - 12) \\
 &= 3x^2 - 3x^2 + 12 = 12 \quad \text{CQFD}
 \end{aligned}$$

$$\begin{aligned}
 5) C &= (2x - 4)^2 - 9 = 4x^2 - 16x + 16 - 9 \\
 C &= 4x^2 - 16x + 7
 \end{aligned}$$

$$\begin{aligned}
 D &= (2x - 1)(2x - 7) = 4x^2 - 14x - 2x + 7 \\
 D &= 4x^2 - 16x + 7
 \end{aligned}$$

$\Rightarrow$ On a bien $C = D$

$$\begin{aligned}
 6) g(x) &= (7 - x)(x - 3) = 7x - 21 - x^2 + 3x \\
 g(x) &= -x^2 + 10x - 21
 \end{aligned}$$

$$\begin{aligned}
 \text{et } 4 - (x - 5)^2 &= 4 - (x^2 - 10x + 25) \\
 &= 4 - x^2 + 10x - 25 = -x^2 + 10x - 21
 \end{aligned}$$

$\Rightarrow$ On a bien $g(x) = 4 - (x - 5)^2$

Corrigé Exercice 13

$$\begin{array}{l|l|l}
 1) f(t) = (t - 2)(2t + 3) & 2) f(-n) = 2(-n)^2 - (-n) - 6 & 3) f(2y) = 2(2y)^2 - (2y) - 6 \\
 f(t) = 2t^2 + 3t - 4t - 6 & f(-n) = 2n^2 + n - 6 & f(2y) = 2 \times 4y^2 - 2y - 6 \\
 f(t) = 2t^2 - t - 6 & & f(2y) = 8y^2 - 2y - 6
 \end{array}$$

$$\begin{aligned}
 4) f(a + 2) &= 2(a + 2)^2 - (a + 2) - 6 \\
 f(a + 2) &= 2(a^2 + 4a + 4) - a - 2 - 6 \\
 f(a + 2) &= 2a^2 + 8a + 8 - a - 8 \\
 f(a + 2) &= 2a^2 + 7a
 \end{aligned}$$

Corrigé Exercice 14

1) On développe :

$$\begin{aligned}
 f(x) &= 2(x - 3)^2 - (x - 3)(4x - 4) \\
 f(x) &= 2(x^2 - 6x + 9) - (4x^2 - 16x + 12) \\
 f(x) &= 2x^2 - 12x + 18 - 4x^2 + 16x - 12 \\
 f(x) &= -2x^2 + 4x + 6
 \end{aligned}$$

$$\begin{aligned}
 b) -2(x - 3)(x + 1) &= -2(x^2 - 2x - 3) \\
 &= -2x^2 + 4x + 6
 \end{aligned}$$

On a bien $f(x) = -2(x - 3)(x + 1)$

$$\begin{aligned}
 c) 8 - 2(x - 1)^2 &= 8 - 2(x^2 - 2x + 1) \\
 &= 8 - 2x^2 + 4x - 2 \\
 &= -2x^2 + 4x + 6
 \end{aligned}$$

On a bien $f(x) = 8 - 2(x - 1)^2$

2) a) $f(x) = -2(x-3)(x+1)$	$f(x) = -2x^2 + 4x + 6$	$f(x) = 8 - 2(x-1)^2$
$f(0) = -2 \times (0-3) \times (0+1)$	$f(0) = -2 \times 0^2 + 4 \times 0 + 6$	$f(0) = 8 - 2 \times (0-1)^2$
$f(0) = 6$	$f(0) = 6$	$f(0) = 8 - 2 \times 1 = 6$

b) c'est la forme 2 la plus adaptée au calcul de l'image de 0 par f (aucun calcul)

3) a) $f(x) = -2(x-3)(x+1)$	$f(x) = -2x^2 + 4x + 6$	$f(x) = 8 - 2(x-1)^2$
$f(3) = -2 \times (3-3) \times (3+1)$	$f(3) = -2 \times 3^2 + 4 \times 3 + 6$	$f(3) = 8 - 2 \times (3-1)^2$
$f(3) = -2 \times 0 \times 4$	$f(3) = -2 \times 9 + 12 + 6$	$f(3) = 8 - 2 \times 2^2$
$f(3) = 0$	$f(3) = -18 + 18 = 0$	$f(3) = 8 - 2 \times 4$
		$f(3) = 8 - 8 = 0$

b) c'est la forme 1 la plus adaptée au calcul de l'image de 3 par f (une des parenthèses s'annule)

4) a) $f(x) = -2(x-3)(x+1)$	$f(x) = -2x^2 + 4x + 6$	$f(x) = 8 - 2(x-1)^2$
$f(1) = -2 \times (1-3) \times (1+1)$	$f(1) = -2 \times 1^2 + 4 \times 1 + 6$	$f(1) = 8 - 2 \times (1-1)^2$
$f(1) = -2 \times (-2) \times 2$	$f(1) = -2 \times 1 + 4 + 6$	$f(1) = 8 - 2 \times 0$
$f(1) = +8$	$f(1) = -2 + 10 = 8$	$f(1) = 8$

b) c'est la forme 3 la plus adaptée au calcul de l'image de 1 par f (la parenthèse s'annule)