Corrections Savoir Sag. 1

Corrigé Exercice 1

1) a) $u_0 = 2 \times 0 + 1 = \mathbf{1}$; $u_1 = 2 \times 1 + 1 = \mathbf{3}$ et $u_2 = 2 \times 2 + 1 = \mathbf{5}$

b) $u_{10} = 2 \times 10 + 1 = \mathbf{21}$

2) a) $w_0 = (-1)^0 = \mathbf{1}$; $w_1 = (-1)^1 = \mathbf{-1}$
$w_2 = (-1)^2 = \mathbf{1}$ et $w_3 = (-1)^3 = \mathbf{-1}$

b) de façon générale, les puissances paires de -1 donne 1 et les puissances impaires, -1 $\Rightarrow$ Donc $w_{15} = \mathbf{-1}$ et $w_{100} = \mathbf{1}$

3) a) $v_2 = \frac{3-2^2}{2-1} = \mathbf{-1}$ $v_3 = \frac{3-3^2}{3-1} = \frac{-6}{2} = \mathbf{-3}$ et $v_4 = \frac{3-4^2}{4-1} = \frac{\mathbf{-13}}{\mathbf{3}}$ **b)** $v_{21} = \frac{3-21^2}{21-1} = \frac{\mathbf{-438}}{\mathbf{20}} = -\frac{\mathbf{219}}{\mathbf{10}}$

4) a) $\varepsilon_1 = \frac{1}{1+\frac{1}{1}} = \frac{\mathbf{1}}{\mathbf{2}}$ $\varepsilon_2 = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{\mathbf{2}}{\mathbf{3}}$

$\varepsilon_3 = \frac{1}{1+\frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{\mathbf{3}}{\mathbf{4}}$ $\varepsilon_4 = \frac{1}{1+\frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{\mathbf{4}}{\mathbf{5}}$

b) $\varepsilon_9 = \frac{1}{1+\frac{1}{9}} = \frac{1}{\frac{10}{9}} = \frac{\mathbf{9}}{\mathbf{10}}$

c) $\varepsilon_n = \frac{1}{1+\frac{1}{n}} = \frac{1}{\frac{n}{n}+\frac{1}{n}} = \frac{1}{\frac{n+1}{n}} = \frac{\boldsymbol{n}}{\boldsymbol{n+1}}$ **CQFD**

Corrigé Exercice 2

a) $u_1 = 2u_0 + 1 = 2 \times 3 + 1 = \mathbf{7}$
$u_2 = 2u_1 + 1 = 2 \times 7 + 1 = \mathbf{15}$
$u_3 = 2 \times 15 + 1 = \mathbf{31}$ et $u_4 = 2 \times 31 + 1 = \mathbf{63}$

b) $v_3 = v_2^2 + \frac{1}{v_2+1} = 3^2 + \frac{1}{3+1} = 9 + \frac{1}{4} = \frac{\mathbf{37}}{\mathbf{4}}$
et $v_4 = \left(\frac{37}{4}\right)^2 + \frac{1}{\frac{37}{4}+1} = \frac{1369}{16} + \frac{1}{\frac{41}{4}} = \frac{1369}{16} + \frac{4}{41} = \frac{\mathbf{56193}}{\mathbf{656}}$

e) $C_2 = \sqrt{c_1^2 + 2} = \sqrt{5^2 + 2} = \sqrt{\mathbf{27}}$
et $C_3 = \sqrt{\left(\sqrt{27}\right)^2 + 2} = \sqrt{27 + 2} = \sqrt{\mathbf{29}}$

f) $d_2 = 3d_1 - 2d_0 = 3 \times (-2) - 2 \times 2 = \mathbf{-10}$
$d_3 = 3d_2 - 2d_1 = 3 \times (-10) - 2 \times (-2)$
$= \mathbf{-26}$
donc $d_4 = 3 \times (-26) - 2 \times (-10) = \mathbf{-58}$

c) $s_2 = \frac{3s_1+1}{s_0-2} = \frac{3\times5+1}{3-2} = \mathbf{16}$ et $s_3 = \frac{3s_2+1}{s_1-2} = \frac{3\times16+1}{5-2} = \frac{\mathbf{49}}{\mathbf{3}}$ donc $s_4 = \frac{3\times\frac{49}{3}+1}{16-2} = \frac{50}{14} = \frac{\mathbf{25}}{\mathbf{7}}$

d) $a_1 = 2 - 3a_0 = 2 - 3 \times (-10) = \mathbf{32}$ **;** $a_2 = 2 - 3 \times 32 = \mathbf{-94}$ et $a_3 = 2 - 3 \times (-94) = \mathbf{284}$

Corrigé Exercice 3

1) a) $f(x) = \frac{\mathbf{3x-7}}{\mathbf{x+3}}$ b) $f(x) = \mathbf{3x^2 - 5x + 1}$

2) a) $g(x) = \mathbf{5x^2 - 3x + 5}$ b) $g(x) = \frac{\mathbf{7}}{\boldsymbol{x}}$

c) Attention, il faut avoir u_{n+1} en fonction de u_n et non l'inverse
donc $u_n = 2u_{n+1} - 5$ $\Leftrightarrow u_n + 5 = 2u_{n+1}$ $\Leftrightarrow$ $u_{n+1} = \frac{u_n+5}{2}$ et $g(x) = \frac{\mathbf{x+5}}{\mathbf{2}}$

Corrigé Exercice 4

1) **a)** $u_{n+1} = \frac{(n+1)-1}{(n+1)+3} = \boldsymbol{\frac{n}{n+4}}$

b) $u_{n-1} = \frac{(n-1)-1}{(n-1)+3} = \boldsymbol{\frac{n-2}{n+2}}$

c) $u_n + 1 = \frac{n-1}{n+3} + 1 = \frac{n-1+n+3}{n+3} = \boldsymbol{\frac{2n+2}{n+3}}$

d) $u_n - 1 = \frac{n-1}{n+3} - 1 = \frac{n-1-(n+3)}{n+3} = \boldsymbol{\frac{-4}{n+3}}$

2) **a)** $u_{n+2} = 5u_{n+1} + 3$ **b)** $u_n = 5u_{n-1} + 3$

c) $u_n - 1 = 5u_{n-1} + 3 - 1 = 5u_{n-1} + 2$

d) $u_{n-1} = 5u_{n-2} + 3$

1) $v_{n+1} = 2(n+1)^2 - 3 = 2(n^2 + 2n + 1) - 3$
$= \boldsymbol{2n^2 + 4n - 1}$

$v_{n-1} = 2(n-1)^2 - 3 = 2(n^2 - 2n + 1) - 3$
$= \boldsymbol{2n^2 - 4n - 1}$

$v_n + 1 = 2n^2 - 3 + 1 = \boldsymbol{2n^2 - 2}$

et $v_n - 1 = 2n^2 - 3 - 1 = \boldsymbol{2n^2 - 4}$

2) $a_{n+1} = \boldsymbol{3a_n^2 - 2a_n}$

$a_{n-1} = \boldsymbol{3a_{n-2}^2 - 2a_{n-2}}$

et $a_n + 1 = \boldsymbol{3a_{n-1}^2 - 2a_{n-1} + 1}$

Corrigé Exercice 5

a. $u_1 = 2(u_0 - 3)^2 = 2\times(7-3)^2 = 2\times 4^2 = \boldsymbol{32}$
$u_2 = 2\times(32-3)^2 = 2\times 29^2 = \boldsymbol{1\,982}$
$u_3 = 2\times(1982-3)^2 = 2\times 1979^2 = \boldsymbol{7\,832\,882}$

b. $u_1 = 2(u_0 - 3)^2 = 2\times(2-3)^2 = 2\times(-1)^2 = \boldsymbol{2}$
$u_2 = 2\times(2-3)^2 = 2\times(-1)^2 = \boldsymbol{2}$
$u_3 = 2\times(2-3)^2 = 2\times(-1)^2 = \boldsymbol{2}$
… Ce qui s'appelle une suite stationnaire ! Comme quoi, il est prouvé l'importance du 1er terme !!!

Corrigé Exercice 6

1) Suite des nombres impairs $\Rightarrow$ $\boldsymbol{v_n = 2n + 1}$

2) Suite des carrés parfaits $\Rightarrow$ $\boldsymbol{w_n = n^2}$

3) Suite des inverses des nombres entiers non nuls $\Rightarrow$ $\boldsymbol{s_n = \frac{1}{n}}$

Corrigé Exercice 7

1) a) $\begin{cases} u_{n+1} = u_n + 3 \\ u_0 = 1 \end{cases}$ **b)** $\begin{cases} u_{n+1} = u_n - 5 \\ u_1 = 100 \end{cases}$ **c)** $\begin{cases} u_{n+1} = 3u_n \\ u_0 = 1 \end{cases}$

d) $\begin{cases} u_{n+1} = u_n + 2n + 1 \\ u_1 = 2 \end{cases}$ **e)** $\begin{cases} u_{n+1} = u_n + u_{n-1} \\ u_0 = 1 \text{ et } u_1 = 1 \end{cases}$ **f)** $\begin{cases} u_{n+1} = (n+1)u_n \\ u_0 = 2 \end{cases}$

2) **a)** $u_{n+1} = 4(n+1) - 1 = 4n + 4 - 1 = (4n-1) + 4 = \boldsymbol{u_n + 4}$

b) $u_{n+1} = (n+1)^2 = n^2 + 2n + 1 = \boldsymbol{u_n + 2n + 1}$

c) $u_{n+1} = 3^n$ et donc $u_n = 3^{n-1} \Rightarrow u_{n+1} = 3^n = 3 \times 3^{n-1} = \boldsymbol{3u_n}$

Corrigé Exercice 8

1) $u_{n+1} = -2 + 5(n + 1) = -2 + 5n + 5 = \mathbf{5n + 3}$ et $w_{n+1} = 4\times3^{n+1} = 4\times3\times3^n = \mathbf{12\times3^n}$

2) Plusieurs méthodes au choix :
1ère méthode : $u_{n+1} - u_n = (5n + 3) - (-2 + 5n) = 3 + 2 = 5 \Leftrightarrow u_{n+1} = u_n + 5$ CQFD
2ème méthode : $u_{n+1} = -2 + 5(n + 1) = (-2 + 5n) + 5 = u_n + 5$ CQFD
3ème méthode : $u_n + 5 = (-2 + 5n) + 5 = -2 + (5n + 5) = -2 + 5(n + 1) = u_{n+1}$ CQFD

Par ex, méthode 3… $3\times w_n = 3\times(4\times3^n) = 4\times(3\times3^n) = 4\times3^{n+1} = w_{n+1}$ $\Rightarrow$ CQFD again

3) Pour u_1 ; w_2 ; u_1 et w_2 la relation de récurrence est la plus simple… Pour u_9 et w_9 ce sont les formules explicites (à moins que vous vouliez vous amuser à calculer les 7 termes précédents…)
$u_1 = -2 + 5 = 3$; $u_2 = 3 + 5 = 8$ et $u_9 = -2 + 5\times9 = 43$
$w_1 = 4\times3 = 12$; $w_2 = 3\times12 = 36$ et $w_9 = 4\times3^9 = 78\,732$